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Gluon Distribution Functions for Very Large Nuclei at Small Transverse Momentum

Larry McLerran and Raju Venugopalan
Theoretical Physics Institute
School of Physics and Astronomy
University of Minnesota
Minneapolis, MN 55455

Abstract

We show that the gluon distribution function for very large nuclei may be computed for small transverse momentum as correlation functions of an ultraviolet finite two dimensional Euclidean field theory. This computation is valid to all orders in the density of partons per unit area, but to lowest order in α_s. The gluon distribution function is proportional to $1/x$, and the effect of the finite density of partons is to modify the dependence on transverse momentum for small transverse momentum.

1 Introduction

In a previous paper, we argued that in a limited range of transverse momentum, for small values of Bjorken x, quark and gluon distributions functions for very large nuclei might be evaluated as the solution of a weakly coupled many body theory[1]. This result relied heavily on the technology of light cone quantization.[2] - [5] Specifically, when a parameter

$$\mu^2 = 1.1 A^{1/3} fm^{-2}\,, \tag{1}$$

corresponding to the density of charge squared fluctuations per unit area, is $\mu^2 >> \Lambda^2_{QCD}$, then the strong coupling is $\alpha_s(\mu^2) << 1$. When the Bjorken x is $x << A^{-1/3}$, it is then valid to replace the valence quarks by delta functions of charge along the light cone. For $\alpha_s \mu^2 << q_t^2 << \mu^2$, and in this range of x, we computed the gluon distribution function to lowest order in weak coupling to be

$$\frac{1}{\pi R^2} \frac{dN}{dx d^2 q_t} = \frac{\alpha_s \mu^2 (N_c^2 - 1)}{\pi^2} \frac{1}{x q_t^2}\,. \tag{2}$$

The gluon distribution function per unit area was precisely the Weizsacker–Williams distribution function for gluons scaled by the density of charge squared fluctuations per unit area. The physical picture corresponding to the above formula is that the Weizsacker–Williams distribution is generated by random fluctuation in the charge per unit area, and is similar in spirit although different in origin than pictures used to describe nucleus–nucleus scattering.[6]-[8]

The approximation of small x guaranties that the central region gluons see a source of valence quarks which are of a much smaller size than a typical gluon wavelength as measured in a frame co–moving with the gluon distribution at that value of Bjorken x. We are therefore in the deeply screened region. In this kinematic region, Lipatov enhancements of the gluon distribution function are expected to modify the Bjorken x dependence of the distribution function and take $1/x \rightarrow 1/x^{1+C\alpha_s}$ where C is some constant[9]. If such a small x enhancement actually

occurs, then the weak coupling expansion which is allowed will only be formal since $\alpha_s(\mu^2)\ln(1/x)$ will not be small. It is not yet clear whether this enhancement actually takes place in the deeply screened small x region we are interested in. If it does, although the coupling constant is weak, one will have to find a way of systematically including the effects of this enhancement.

We will not address the small x enhancement in this paper. We shall instead turn to another aspect of the problem–which is, computing the distribution functions in the small q_t region. We will here work to lowest (formal?) order in α_s but to all orders in $\alpha_s^2\mu^2$. We will show that the correlation function which gives the gluon distribution function can be expressed as a two dimensional Euclidean correlation function of an ultraviolet finite field theory. We will find that summing to all orders in $\alpha_s^2\mu^2$ modifies the q_t distribution function. However, to all orders in this expansion, the gluon distribution function is proportional to $1/x$.

Before deriving these results, we shall first briefly review the results of our earlier paper. We recall that in the small x region, the valence quarks are Lorentz contracted to a smaller distance scale than that of the wavelength of the co–moving gluon. Therefore the valence quarks may be treated as being approximately delta functions along the light cone. We can ignore valence quark recoil, so long as the gluons being emitted are soft and so long as the coupling is weak. In this limit, the valence quarks may be treated as sources of charge.

The valence quarks may also be taken to be uniformly distributed in transverse space for sufficiently large nuclei. In this case, the scale of variation of the nuclear valence quark distribution along the transverse direction can be made large compared to the typical hadronic distance scales.

The problem therefore is to compute the distribution function of gluons in the presence of static sources of color charge localized along the light cone and uniform

in transverse space. The external current due to the source may be represented as

$$J_a^\mu = \delta^{+\mu} Q_a(x^+, x_t)\delta(x^-)\,. \tag{3}$$

It was shown in our first paper that the nucleus can be broken up into regions of transverse spatial extent such that the number of valence quarks in each region is large. This allows us to treat the sources of charge as classical. We also showed that the dominant contribution to the ground state wavefunction came from states which had large fluctuations around zero color charge, but where the fluctuations were small compared to the total amount of charge in each transverse spatial region. In this limit, the fluctuations are gaussian.

The $A^{1/3}$ dependence of the gluon distribution function in this screened region follows from the above arguments. While the color charge is screened so that the average color charge is of order $\sqrt{N}$, where N is the total color charge in each spatial region, the coherence of the color field makes the gluon density of order N. The number of valence quarks per unit transverse area goes as $A^{1/3}$. The gluon density therefore goes as $A^{1/3}$–up to corrections due to the logarithmic dependence on A of the coupling constant.

The problem is therefore a simple one: If we want to compute a ground state correlation function, we can do it by the path integral

$$\int [d\rho][dA]\ e^{-\frac{1}{2\mu^2}\rho^2}\ e^{\{iS[A]+iA_+\rho\}} \tag{4}$$

that is, we just integrate the path integral for fixed charge around a Gaussian fluctuating charge at each point in space. We must therefore compute correlation functions in a stochastic background field.

The approximation that we may treat the source as classical is only true in the limit where the spatial regions we are looking at have a large number of quarks in them. In our first paper, we argued that this requires that

$$q_t^2 << \mu^2 \tag{5}$$

We will assume that this is also the case in this paper.

In the previous paper, we evaluated the ground state gluon distribution function in the perturbative region where both α_s and $\alpha_s^2\mu^2$ were treated as small parameters. This latter condition forced $\alpha_s^2\mu^2 << q_t^2$. In this paper, we will relax this condition and set up the computation of the gluon distribution function in the soft region.

2 The Classical Problem

We first turn to the problem of computing the solution of the classical problem for the gluon field in the presence of a source which is a delta function along the light cone. The equation of motion is

$$D_\mu F^{\mu\nu} = gJ^\nu \,, \tag{6}$$

where J is the classical light cone source. We will work in light cone gauge where $A_- = -A^+ = 0$.

The current J^μ only has components along the + direction and is proportional to a delta function of x^-. There exists a solution of the equations of motion for this problem, where the longitudinal component A_+, which is not zero by a gauge, vanishes by the equations of motion

$$A_+ = -A^- = 0 \,. \tag{7}$$

The only non–zero components of the field strength therefore are the transverse components which we require to be of the form

$$A_i(x) = \theta(x^-)\alpha_i(x_t) \,, \tag{8}$$

where $\theta(x^-) = +1$ for $x^- > 0$ and $\theta(x^-) = 0$ for $x^- < 0$. This function is nonvanishing only when $x^- > 0$, which since $x^- = (t-x)/\sqrt{2}$ is equivalent to $x < t$. This is what we expect for a classical field generated by a source traveling

close to the speed of light with $x = t$. For $x > t$, the source has not yet arrived, and for $x < t$ the source should produce a field.

Using the definition of F^{i+} in terms of A^i

$$F^{i+} = \delta(x^-)\alpha_i \,. \tag{9}$$

If we further require that

$$F^{ij} = 0 \,, \tag{10}$$

(where i and j are transverse components), we see that we have a solution of the equations of motion so long as

$$\nabla \cdot \alpha = g\rho(x_t) \,. \tag{11}$$

Here ρ is the surface charge density associated with the current J. There is no dependence on x^- because we have factored out the delta function. The dependence on x^+ goes away because the extended current conservation law,

$$\partial_+ Q^a + if^{abc} A^b_+ Q^c = 0 \,, \tag{12}$$

is simplified by the solution of the field equations in Eq. (7) to read

$$\partial_+ Q^a = 0 \,. \tag{13}$$

Hence,

$$Q^a(x^+, x_t) = \rho^a(x_t) \,. \tag{14}$$

The condition that $F^{ij} = 0$ is precisely the condition that the field α is a gauge transform of the vacuum field configuration for a two dimensional gauge theory. The requirement that $\nabla \cdot \alpha = g\rho$, is a condition that fixes the gauge. For such a field configuration, the light cone Hamiltonian

$$P^- = 0 \,. \tag{15}$$

This is the precise analog of what we found in our previous paper for the Weizsacker–Williams distribution around an electron.

The field configuration which is a gauge transform of the vacuum field configuration for a two dimensional field theory may be written as

$$\tau \cdot \alpha_i = -\frac{1}{g} U \frac{1}{i} \nabla_i U^\dagger \, . \tag{16}$$

We have not been able to construct explicit solutions for the above equation for arbitrary dependence of the surface charge density on x_t.

Note that the field configuration which solves this problem only has a nontrivial dependence on x_t. The dependence on x^-, as shown in Eq. (8) is only through a step function, and upon Fourier transforming gives only a factor of $1/k^+$. As we will see in the next section, the distribution functions associated with this field therefore, to all orders in $\alpha_s^2\mu^2$, have the general form

$$\frac{1}{\pi R^2} \frac{dN}{dx d^2k_t} = \frac{(N_c^2-1)}{\pi^2} \frac{1}{x} \, \frac{1}{\alpha_s} H(k_t^2/\alpha_s^2\mu^2) \, . \tag{17}$$

In our previous paper, we showed explicitly that in the weak coupling limit we obtained the simple result

$$H(y) = 1/y \, , \tag{18}$$

for

$$1 << y << 1/\alpha_s^2 \, . \tag{19}$$

We see here that this result is true only in the weak coupling limit where $\alpha_s^2\mu^2 << k_t^2$. The range is larger than that used in our previous work. Note also that this result is parallel to that of finite temperature field theory where there is a non-perturbative length scale, the magnetic screening length, which is $\lambda \sim 1/\alpha_s T$ where T is the temperature.

3 Computing Correlation Functions

To compute correlation functions associated with our classical solutions, we must integrate over all ρ. This is equivalent to integrating over the transverse field with the constraint that the field must be a pure gauge, that is,

$$\int [d\alpha] \; e^{-(\nabla\alpha)^2/2g^2\mu^2} \; \delta(F^{12}) \,. \tag{20}$$

We can perform the integration over the transverse field α as integrations over unitary matrices in the standard way in which one goes to a latticization of a gauge theory. The integration measure becomes

$$\int [dU] \exp\left(-\frac{1}{g^4\mu^2} tr \left(\nabla(U \frac{1}{i} \nabla U^\dagger)\right)^2\right) \tag{21}$$

We see that the measure for this theory is that for a two dimensional Euclidean field theory. The spatial variables are those of the two dimensional transverse space of the original theory. This theory is ultraviolet finite because of the fact the Lagrangean is fourth order in derivatives. The expansion parameter for the theory is is $\alpha_s^2\mu^2/k_t^2$.

The above analysis ignores Fadeev-Popov determinants which are generated in transforming from the integration over ρ to the integration over α and finally to that over U. To properly define the two dimensional theory for purposes of Monte-Carlo simulation, one may have to investigate these determinants. For our purpose, which is to study the scaling behavior of expectation values, we will not take into account these determinants. The determinant for both integral representations in terms of α or in terms of U is easy to compute, and is the same for both representations. It is the determinant of $\vec{\nabla} \cdot \vec{D}$ where $\vec{D}$ is the covariant derivative associated with the field α. This leads in two dimensions to an ultraviolet finite modification of the above measure.

As discussed in the previous section and more explicitly in our earlier paper, when the relevant momentum scale is $k_t^2 >> \alpha_s^2\mu^2$, the theory is in the weak coupling

region and may be evaluated perturbatively . For smaller values of k_t^2, we are in the strong coupling phase of the theory. In this phase of the theory, we expect that there should be no long range order. Correlation functions of x_t should die exponentially at large distances, or alternatively the Fourier transform of correlation functions should go like a constant for small momentum. We will see that this guarantees the finiteness of the gluon distribution functions for small momentum.

It should be easy to compute correlation functions for the above action using lattice Monte Carlo methods. The theory is two dimensional which should make possible the use of large grids. The theory is Euclidean so that all quantities of physical interest are computable as Euclidean correlation functions. The theory is ultraviolet finite, so that there should be no problems extrapolating to the continuum limit.

Finally, the correlation function for the computation of the transverse momentum dependence of the structure function must be determined. This may be simply evaluated to be

$$D(k_t) = \frac{1}{\alpha_s} H(k_t) \tag{22}$$

where D is the propagator for the two dimensional theory

$$(2\pi)^2 \delta^2(k_t - q_t) \delta_{ij} D(k_t) =< \alpha_i \alpha_j > \tag{23}$$

The expression for the propagator can be easily rexpressed in terms of the link variables U. As claimed, the small k_t behavior of the gluon distribution function is related to the asymptotics in coordinate space for the propagator of the two dimensional theory. Large distances correspond to strong coupling, which in turn corresponds to a lack of correlations signalled by an exponential fall off. We therefore expect that at small k_t, the function $H(k_t)$ will be finite; the gluon distribution function will be non–singular.

4 Summary and Conclusions

We have seen that the infrared behavior of the gluon distribution function for small k_t is determined by solving a two dimensional field theory. The relevant fields are gauge transforms of two dimensional vacuum configurations. The light cone Hamiltonian vanishes for such configurations. The field theory for the correlation functions of interest is finite and involves integrating over gauge transforms of the field with a specified weight function.

It is clear that the gluon propagator and light quark propagator in the presence of such a background field configuration should be quite simple. The solutions of the small fluctuation equations are just gauge transforms of free field solutions. To construct the propagator one must join solutions across the discontinuity in x^- generated by the source of charge. The study of these propagators will be the subject of later analyses.

It is also clear that the infrared behavior of these correlation functions is computable as a lattice Monte Carlo simulation. Realistically, it appears that the desired accuracy might be obtained.

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